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Path Tracking by an Accelerometer with α-β-γ and α-β-γ-δ Filters

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In this paper, we propose path tracking using an accelerometer sensor with α-β-γ and α-β-γ-δ filters. In addition, the error and reliability are evaluated. The static, free-fall motion, circular experiments demonstrate error accumulation with α-β-γ and α-β-γ-δ filters. In the positioning algorithm, the acceleration values are obtained by an accelerometer sensor. In this paper, a series of experimental tests was conducted for the comparison between α-β-γ and α-β-γ-δ filters. The results show that more accurate path tracking could be achieved if tracking were under feedback control using α-β-γ-δ filter to α-β-γ filter.

1. Introduction

Path tracking is a basic function among mobile robot or autonomous vehicle control systems. The objective of path tracking is to follow a previously recorded or planned path with an accelerometer. Filters are important tools for path tracking systems to estimate position. There are many solutions developed to deal this problem, which is known as the least squares estimation,[1] Monte Carlo filter, Kalman filter,[2] extended Kalman filter,[3] α-β filter,[4] α-β-γ filter,[5] α-β-γ-δ filter,[6–8] and others.[9,10] Meanwhile, filters are also introduced to reduce error accumulation. G. Antonelli *et al*.[9] realized a path tracking system using an accelerometer and gyroscope and employed a Kalman filter smoothing algorithm to overcome the error accumulation problem. Liu and Pang[10] developed path tracking using a low-cost accelerometer.

These filters have two parameters: the measurement noise variance and the process noise variance. The measurement noise variance can be obtained from the sensor specifications. The filter is tuned using the process noise variance. Nevertheless, error accumulation occurs during path tracking. Some methods have been recently proposed to reduce the error accumulation.

Path tracking was applied using an α-β-γ filter to reduce error accumulation in this study. But this filter cannot track acceleration effectively. To solve this problem, a new α-β-γ-δ filter was introduced. In this study, the path tracking was applied using a monolithic accelerometer sensor to check the error and reliability of the system. The acceleration was acquired, and then the positioning algorithm was performed. The acceleration data was integrated as parameters for both

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filters. The path tracking errors between the α-β-γ and α-β-γ-δ filters were then compared. The results illustrate that error accumulation of the direct integral in the positioning algorithm with an α-β-γ-δ filter was effectively improved and reduced.

2. System Architecture

The architecture of path tracking is illustrated in Fig. 1. The ADXL330 used in this research was a small, low-power, and complete 3-axis accelerometer with signal conditioned voltage outputs. It could measure acceleration with a minimum full-scale range of ±3*g*. The outputs were digital signals and were measured directly through a microcontroller such as a PIC16F877A. The Bluetooth modules communicated with integration of tilt or obtained the acceleration, while the position was roughly estimated by double integration of the acceleration data. The acceleration was communicated to a personal computer for position estimation through Bluetooth wireless modules.

3. Mathematical Model

Consider a one dimensional position, velocity acceleration, jerk discrete-time target motion model:

$$\begin{bmatrix} x(k+1) \\ \dot{x}(k+1) \\ \ddot{x}(k+1) \\ \dddot{x}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 & \frac{1}{6}T^3 \\ 0 & 1 & T & \frac{1}{2}T^2 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ \dot{x}(k) \\ \ddot{x}(k) \\ \dddot{x}(k) \end{bmatrix} + \begin{bmatrix} \frac{1}{2}T^2 \\ T \\ 1 \\ 0 \end{bmatrix} w(k). \tag{1}$$

When the motion is not considered by a jerk model, the equation can be simplified as follow:

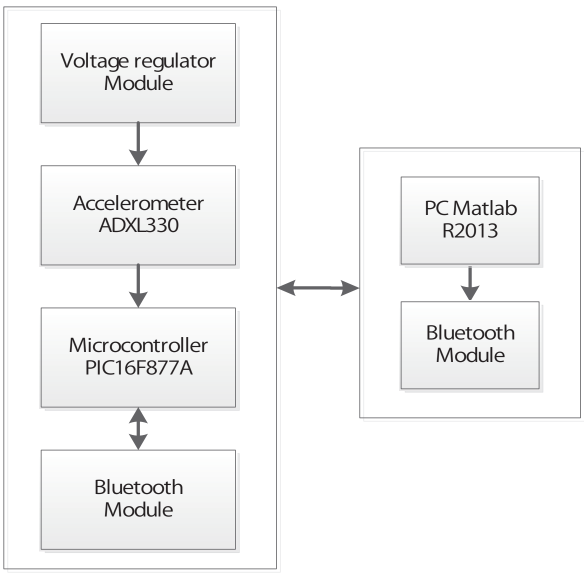

Fig. 1. System architecture.

$$\begin{bmatrix} x(k+1) \\ \dot{x}(k+1) \\ \ddot{x}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{1}{2}T^2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ \dot{x}(k) \\ \ddot{x}(k) \end{bmatrix} + \begin{bmatrix} \frac{1}{2}T^2 \\ T \\ 1 \end{bmatrix} w(k), \tag{2}$$

where $x(k)$ is the position at time k, $v(k) = \dot{x}(k)$ is the velocity at time k, $a(k) = \ddot{x}(k)$ is acceleration at time k, $j(k) = \dddot{x}(k)$ is the jerk state at time k, T is the time step or time increment, and $w(k)$ is the measurement noise of the accelerometer sensor at time k.

The α-β-γ filter is a third-order tracker capable of predicting the object's next position and velocity by tracking the current and past positions and velocities.

The equations are given by:

$$x_p(k+1) = x_s(\mathrm{k}) + Tv_s(k) + \frac{1}{2}T^2[a_s(k) + w(k)], \tag{3}$$

$$v_p(k+1) = v_s(k) + T[a_s(k) + w(k)], \tag{4}$$

$$a_p(k+1) = a_s(k) + w(k), \tag{5}$$

where subscripts "p" and "s" denote the predicted and smoothed values, respectively.

The parameters are derived based on the previous prediction and the weighted innovation in the following:

$$x_s(k) = x_p(k) + \alpha[x_o(k) - x_p(k)], \tag{6}$$

$$v_s(k) = v_p(k) + \frac{\beta}{T}[x_o(k) - x_p(k)], \tag{7}$$

$$a_s(k) = a_p(k-1) + \frac{\gamma}{2T^2}[x_o(k) - x_p(k)], \tag{8}$$

where the subscript "o" denotes the exact value in mathematics and signal processing.

Jury's stability test[11] yields the constraints on α, β, and γ parameters for the α-β-γ filter as follows. It is also used to find the stability domain for the characteristic polynomial (CP) of Eq. (6).

$$0 < \alpha < 2 \tag{9}$$

$$0 < \beta < \frac{13}{6}(4 - 2\alpha) \tag{10}$$

$$0 < \gamma < \frac{4\alpha\beta}{2 - \alpha} \tag{11}$$

To improve tracking accuracy, the mathematical equations of a fourth-order α-β-γ-δ filter target tracker included in predicting acceleration are given by:

$$x_p(k+1) = x_s(k) + Tv_s(k) + \frac{1}{2}T^2a_s(k) + \frac{1}{6}T^3j_s(k) + \frac{1}{2}T^2w(k), \tag{12}$$

$$v_p(k+1) = v_s(k) + Ta_s(k) + \frac{1}{2}T^2 j_s(k) + Tw(k), \tag{13}$$

$$a_p(k+1) = a_s(k) + Tj_s(k) + w(k), \tag{14}$$

where

$$x_s(k) = x_p(k) + \alpha[x_o(k) - x_p(k)], \tag{15}$$

$$v_s(k) = v_p(k) + \frac{\beta}{T}[x_o(k) - x_p(k)], \tag{16}$$

$$a_s(k) = a_p(k) + \frac{\gamma}{2T^2}[x_o(k) - x_p(k)], \tag{17}$$

$$j_s(k) = j_p(k-1) + \frac{\delta}{6T^3}[x_o(k) - x_p(k)]. \tag{18}$$

Jury's stability test yields the constraints on α, β, γ, and δ parameters for the α-β-γ-δ filter as follows.

$$0 < \alpha < 2 \tag{19}$$

$$0 < \beta < \frac{13}{6}(4 - 2\alpha) \tag{20}$$

$$0 < \gamma < \frac{4\alpha\beta}{2 - \alpha} \tag{21}$$

$$0 < \delta < 24(2 - \alpha) \tag{22}$$

In this study, a method to search for the adaptive parameter was applied to optimize the design of the α-β-γ and α-β-γ-δ filters. Four evenly spaced elements are created in the interval of ranges for each of α, β, and γ parameters. And six evenly spaced elements are created in the interval of range for δ parameter because of its large range. The α-β-γ filter requires 64 runs to compute as Eqs. (9)–(11). The optimal design minimizes the absolute value of the acceleration error $|\ddot{x}_s(k) - \ddot{x}_o(k)|$ to find the optimal parameter (α^*, β^*, γ^*) for every time step. The absolute value of the acceleration error is a function of α, β, and γ. First, we supply the same parameters β^* and γ^*. Then we use three-point (α^1, α^*, α^2) curve fitting to find the local optimal parameter $\hat{\alpha}$; α^1 and α^2 are the neighboring levels of the α^* parameter.

$$f(\alpha^1, \beta^*, \gamma^*) = \lambda_1 \cdot \alpha^1 \cdot \alpha^1 + \lambda_2 \cdot \alpha^1 + \lambda_3 \tag{23}$$

$$f(\alpha^*, \beta^*, \gamma^*) = \lambda_1 \cdot \alpha^* \cdot \alpha^* + \lambda_2 \cdot \alpha^* + \lambda_3 \tag{24}$$

$$f(\alpha^2, \beta^*, \gamma^*) = \lambda_1 \cdot \alpha^2 \cdot \alpha^2 + \lambda_2 \cdot \alpha^2 + \lambda_3 \tag{25}$$

When the following two conditions are satisfied, we can determine the local optimal parameter

$\hat{\alpha} = -0.5 \times \lambda_2 / \lambda_1$:

$$f(\alpha^*, \beta^*, \gamma^*) \leq f(\alpha^1, \beta^*, \gamma^*) \text{ and } f(\alpha^*, \beta^*, \gamma^*) \leq f(\alpha^2, \beta^*, \gamma^*). \tag{26}$$

Finally we use the same method to find $\hat{\beta}$ and $\hat{\gamma}$.

The *α-β-γ-δ* filter requires 384 runs to compute Eqs. (19)–(22). The optimal design minimizes the absolute value of the jerk error $|\dddot{x}_s(k) - \dddot{x}_o(k)|$ to find the optimal parameters (α^*, β^*, γ^*, δ^*) and the local optimal parameters $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, and $\hat{\delta}$ for every time step.

4. Experiments

A series of experimental tests for the comparison between *α-β-γ* and *α-β-γ-δ* filters was conducted. Three experimental cases (static, free-fall, and circular) are introduced as follows.

4.1 Static experiment

In this case, an accelerometer was used to take samples every 0.0256 s in the static situation. Acceleration noise added to the *X*-, *Y*- and *Z*-axes was 0.0034, 0.0026, and 0.0014 m/s^2, respectively. Table 1 summarizes the path tracking errors in the mean square error (MSE) and the maximum error with an *α-β-γ* filter and an *α-β-γ-δ* filter by taking 400 samples in the static experiment. All MSEs and maximum errors were very small. Above all, the errors resulting from the *α-β-γ-δ* filter were much smaller than those from the *α-β-γ* filter. The maximum errors were 2.8609e−004 m on the *Z*-axis by the *α-β-γ* filter and 2.4517e−004 m on the *X*-axis by the *α-β-γ-δ* filer. These results clearly indicate that the positioning algorithm with the *α-β-γ-δ* filter exhibits a significant improvement for the path tracking problem.

4.2 Free-fall experiment

Theoretically during a free fall, the vertical acceleration is expected to be the magnitude of gravity while the horizontal acceleration is zero. Acceleration noise added to the *X*-, *Y*-, and *Z*-axes was 0.00398, 0.0382, and 0.0430 m/s^2, respectively. Table 2 shows the path tracking errors in the MSE and the maximum error with an *α-β-γ* filter and an *α-β-γ-δ* filter by taking 100 data points in the free-fall experiment. In addition, Fig. 2 shows that the errors resulting from the *α-β-γ-δ* filter

Table 1
Path tracking errors on MSE and maximum error with *α-β-γ* filter and *α-β-γ-δ* filter in static experiment.

α-β-γ filter	MSE	Max. error	CPU time (s)
X-axis	3.1653e−006	2.5165e−004	0.031090
Y-axis	2.5038e−006	1.9288e−004	0.016896
Z-axis	2.7276e−006	2.8609e−004	0.017134

α-β-γ-δ filter	MSE	Max. error	CPU time (s)
X-axis	1.8807e−006	2.4517e−004	0.738975
Y-axis	2.2577e−007	2.7457e−005	0.575638
Z-axis	1.7232e−007	1.9755e−005	0.571654

Table 2
Path tracking errors on MSE and maximum error with *α*-*β*-*γ* filter and *α*-*β*-*γ*-*δ* filter in free-fall experiment.

α-*β*-*γ* filter	MSE	Max. error	CPU time (s)
X-axis	4.9340e−006	1.2367e−004	0.014291
Y-axis	6.4752e−006	2.0805e−004	0.003991
Z-axis	4.7662e−006	1.4640e−004	0.003830

α-*β*-*γ*-*δ* filter	MSE	Max. error	CPU time (s)
X-axis	2.1839e−006	7.3130e−005	0.169337
Y-axis	6.0057e−007	2.0751e−005	0.143121
Z-axis	3.6099e−007	1.3466e−005	0.142003

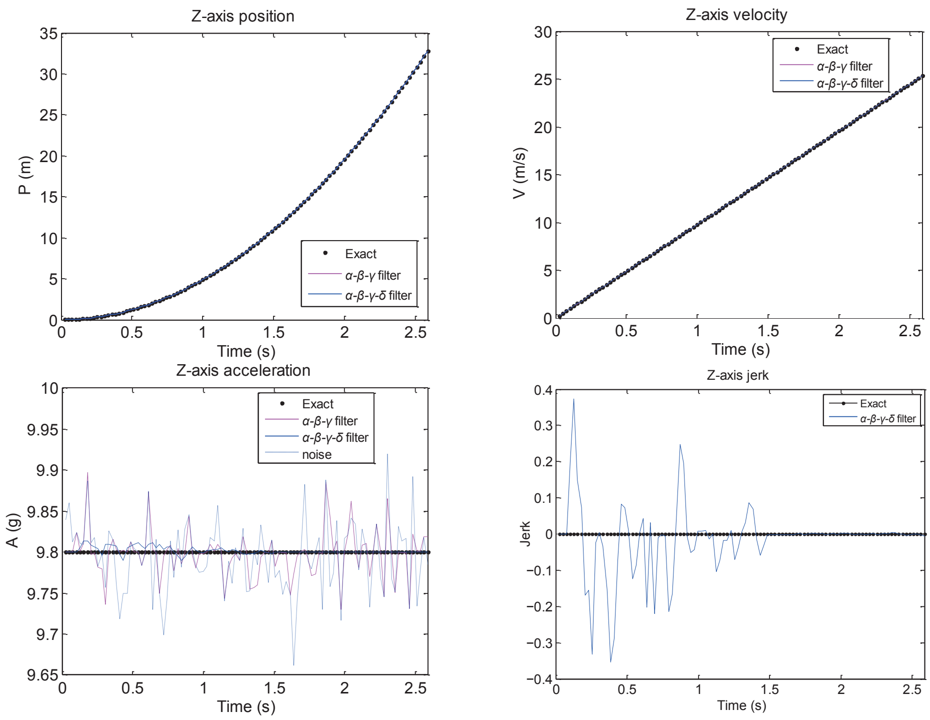

Fig. 2. (Color online) Representative time trajectories on *Z*-axis during free-fall experiment.

were much smaller than those from the *α*-*β*-*γ* filter. Obviously, the process with the *α*-*β*-*γ*-*δ* filter was more accurate than with the *α*-*β*-*γ* filter.

4.3 Circular experiment

For this experiment, the setting of the two-dimensional rotation mechanism means that the centripetal direction was positive along the *X*-axis, the tangent direction was positive along the *Y*-axis, the radius was 0.1 m, and the angular speed was 0.6 rad/s at 5 V. The rotational speed was constant. As in the static experiment, data was taken every 0.0256 s. Table 3 shows the path tracking errors in the MSE and the maximum error with an *α*-*β*-*γ* filter and an *α*-*β*-*γ*-*δ* filter. The

Table 3
Path tracking errors on MSE and maximum error with α-β-γ filter and α-β-γ-δ filter in circular experiment.

α-β-γ filter	MSE	Max. error	CPU time (s)
X-axis	7.4236e−007	9.7878e−005	0.041772
Y-axis	7.5355e−007	1.0160e−004	0.030846
Z-axis	7.2742e−007	1.1186e−004	0.030614
α-β-γ-δ filter	MSE	Max. error	CPU time (s)
X-axis	8.9181e−008	1.6130e−005	1.178518
Y-axis	9.8591e−008	1.4721e−005	1.152375
Z-axis	1.0739e−007	4.1759e−005	0.142003

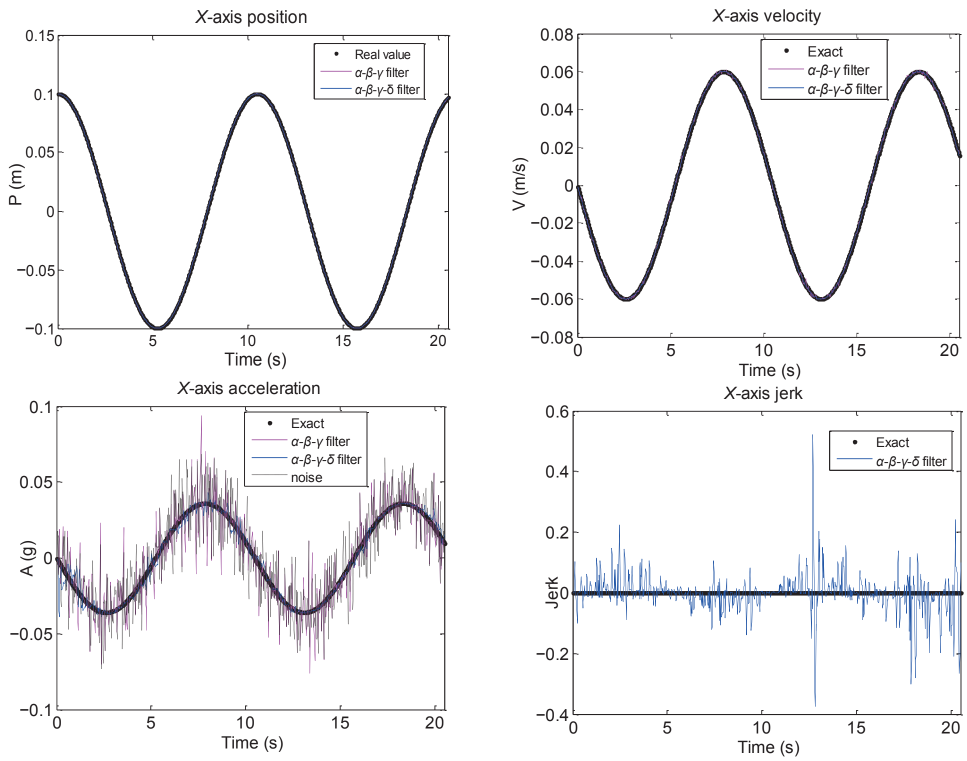

Fig. 3. (Color online) Representative time trajectories on *X*-axis during circular experiment.

results in Fig. 3 show the trends of taking 800 data points with an α-β-γ filter and an α-β-γ-δ filter. In this case, the MSE and maximum error of the α-β-γ-δ filter were smaller than those from the α-β-γ filter in the three experiments. The computational time of the α-β-γ-δ filter were much greater than that of the α-β-γ filter in the three experiments. The results indicate that both the α-β-γ and α-β-γ-δ filters have high convergent efficiency but the α-β-γ-δ filter was still better than the α-β-γ filter.

5. Conclusion

A path tracking system with an α-β-γ filter and an α-β-γ-δ filter was compared and examined in experimental tests. The error and reliability of the path tracking system was evaluated in three different experiments. All MSEs and maximum errors were very small, especially the errors resulting from the *α*-*β*-*γ*-*δ* filter were much smaller than those from the *α*-*β*-*γ* filter in all experiments. There is much evidence to indicate that the *α*-*β*-*γ*-*δ* filter is more efficient than the *α*-*β*-*γ* filter. The *α*-*β*-*γ*-*δ* filter offers a robust prediction technique and reduces the estimate error to a minimum. It could be applied indoors and in restricted environments. It can also be combined with path tracking for greater accuracy and more widespread applications.

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