Sensors and Materials, Vol. 12, No. 8 (2000) 473–490
MYU Tokyo

S & M 0423

Isotropic Piezoresistance in Polycrystalline Silicon for In-plane Shear- and Normal-Stress Gauges

Toshiyuki Toriyama, Yoshihiko Yokoyama and Susumu Sugiyama

Faculty of Science and Engineering, Ritsumeikan University,
1–1–1 Noji-higashi, Kusatsu, Shiga 525–8577, Japan.

(Received May 24, 2000; accepted October 2, 2000)

Key words: piezoresistance, isotropy, polycrystalline silicon, in-plane shear, stress gauge

In this study, an isotropic piezoresistance equation for polycrystalline silicon (poly-Si), focusing particularly on shear piezoresistance in p-type poly-Si, is derived. The total number of independent piezoresistance coefficients to describe the isotropic piezoresistance effect of poly-Si is found to be two. It is also found that, for an isotropic poly-Si Hall-type piezoresistor, the change in resistance is induced only by normal stresses while the change in transverse voltage is induced only by shear stresses. The relationship between the two independent piezoresistance coefficients of the isotropic poly-Si and three independent piezoresistance coefficients of single crystalline Si (single-Si) was derived using a crystallographic orientation averaging (overall averaging) method. Experiments on the isotropic p-type poly-Si Hall-type piezoresistors were carried out under in-plane normal and in-plane shear stress conditions in order to confirm the derived piezoresistance characteristics.

1. Introduction

General piezoresistance effects can be described separately in two cases by means of Hall-type (four-terminal) piezoresistors. The first is a change in resistance due to stress, and this effect can be measured through two terminals in the Hall-type piezoresistor in the case where current and voltage are in the same direction. The second is a change in transverse voltage due to stress and this effect can be measured through four terminals in the Hall-type piezoresistor in the case where the current is orthogonal to the voltage. Pfann and Thurston proposed these concepts about forty years ago.[1]

Kanda first described the piezoresistance effect of a single crystalline silicon (single-Si) Hall-type piezoresistor and its application to pressure sensors in 1982 and 1983.[2,3] Bao and Wang investigated both the theory of and experiments on the single-Si Hall-type piezoresistor and its applications as a pressure sensor.[4] The shear piezoresistance is important for the design of the Hall-type piezoresistors. Kanda first described graphical representation of the shear piezoresistance coefficients of single-Si as a function of crystallographic orientation, and clarified the anisotropy of the shear piezoresistance.[5] He also derived the mathematical relation between three independent piezoresistance coefficients of single-Si with respect to the principal crystallographic orientations, and the shear piezoresistance coefficients appearing in the Hall-type piezoresistors.

In the last two decades, since the pioneering work of Seto[6] in 1976, many reports on the effects of change in resistance due to stress in polycrystalline silicon (poly-Si) have been published.[7–11] Poly-Si has advantages for the fabrication of stress gauges, since the fabrication of isolated junctions is not required and because it can be deposited onto various kinds of insulator-coated substrates. The general piezoresistance effects of poly-Si using Hall-type piezoresistors were first investigated by French and Evans in 1998,[12] both by theory and experiment. They proposed separate models to describe the effects of change in resistance and the transverse voltage due to stresses. In their treatment, the relationship between the piezoresistance coefficients, which contribute to the resistance change and the transverse voltage change, are not clarified. Therefore, determination of the total number of independent coefficients which are necessary to describe the piezoresistance effect in poly-Si, remains a problem. French and Evans also derived the piezoresistance coefficients of poly-Si from that of single-Si when change in resistance due to stress occurs. However, they did not provide any suggestions for this method when change in transverse voltage due to stress occurs.

Bossche and Mollinger investigated change in resistance due to stress for a poly-Si two-terminal piezoresistor both by theory and experiment, and proposed a method of stress measurement appearing in molded IC reliability problems[13] They determined the theoretical number of independent piezoresistance coefficients which are necessary to explain the piezoresistance effect of poly-Si. However, they did not take into account the effect of change in transverse voltage due to stress. Therefore, the total number of independent piezoresistance coefficients is expected to be different from their value when the transverse voltage effect is introduced into their model. They also did not suggest any method for deriving the piezoresistance coefficients of poly-Si from that of single-Si.

Thus, results taken from the literature can be summarized as follows. In the case of single-Si, the effects of change in resistance and transverse voltage due to stress can be described explicitly by the three independent piezoresistance coefficients. However, in the case of poly-Si, investigations of the total number of independent piezoresistance coefficients which are necessary to describe effects of change in resistance and transverse voltage due to stress, are unsatisfactory and thus the problem still remains. The relationship between the piezoresistance coefficients of poly-Si and that of single-Si is also not clarified, except in the special case derived by French and Evans.[9,10,12]

In this study, the relationship between the piezoresistance coefficients of isotropic poly-Si Hall-type piezoresistors which correspond to changes in resistance and transverse voltage due to stress was investigated. We found that the total number of independent piezoresistance coefficients required to describe the isotropic piezoresistance effect of poly-Si is two. As a consequence, for the isotropic poly-Si Hall-type piezoresistor, the change in resistance is induced only by normal stresses and the change in transverse voltage is induced only by shear stresses. The relationship between the two independent piezoresistance coefficients of the isotropic poly-Si and the three independent piezoresistance coefficients of single-Si was derived using a crystallographic orientation averaging (overall averaging) method. Experiments on the isotropic poly-Si Hall-type piezoresistors were carried out under in-plane normal and in-plane shear stress conditions in order to confirm the above-mentioned in-plane piezoresistance characteristics.

2. Overall Averaging Method

Throughout this paper, the Cartesian coordinate system is used and if any index occurs twice in any one term, summation is taken from 1 to 3 (Einstein summation convention). The piezoresistance in polycrystalline semiconductors can be derived by performing overall averaging of single-crystalline values, with respect to the crystallographic orientations (see Appendix A1). The piezoresistance in polycrystalline semiconductors can be expressed as

$$\langle E_i \rangle = \langle \rho_{ij} + \Delta\rho_{ij} \rangle \langle J_j \rangle \tag{1}$$

$$\langle \Delta\rho_{ij} \rangle = \langle \pi_{ijkl} \rangle \langle \sigma_{kl} \rangle, \tag{2}$$

where crystallographic orientation averaging of any quantity is defined by the angular brackets $\langle\ \rangle$, where $\langle E_i \rangle$ (V/m) is the electric field vector, $\langle J_j \rangle$ (A/m^2) is the current density vector, $\langle \rho_{ij} \rangle$ (Ω-m) is the resistivity tensor, $\langle \Delta\rho_{ij} \rangle$ (Ω-m) is the relative change in the resistivity tensor due to applied stress, $\langle \pi_{ijkl} \rangle$ (Ω-m/MPa) is the piezoresistance tensor for a polycrystalline semiconductor, and $\langle \sigma_{kl} \rangle$ (MPa) is the stress tensor.

In the case of isotropy, the calculation of $\langle \pi_{ijkl} \rangle$ in terms of the single-crystalline tensor π_{ijkl} (i.e., random distribution of single crystals) is simple and the result is expressed as (see Appendix A2)

$$\langle \pi_{ijkl} \rangle = \Pi_1 \delta_{ij} \delta_{kl} + \Pi_2 (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \tag{3}$$

where $\Pi_1 = \frac{1}{15}(2\pi_{ppqq} - \pi_{pqpq})$, $\Pi_2 = \frac{1}{30}(3\pi_{pqpq} - \pi_{ppqq})$ and δ_{ij} is the Kronecker delta.

3. Isotropic Piezoresistance Effect in Polycrystalline Semiconductors

By combining of eqs. (1) to (3), the isotropic piezoresistance in polycrystalline semiconductors can be expressed as

$$\langle \Delta\rho_{ij} \rangle = \Pi_1 \delta_{ij} \langle \sigma_{kk} \rangle + 2\Pi_2 \langle \sigma_{ij} \rangle. \tag{4}$$

For practical use in engineering, the tensors Π_1 (Ω-m/MPa) and Π_2 (Ω-m/MPa) can be transformed into coefficients π_1 (1/MPa) and π_2 (1/MPa) by factoring out the resistivity of $\langle\rho\rangle$ (Ω-m) (Masons convention[14]):

$$\Pi_1 = \langle\rho\rangle\pi_1 \tag{5}$$

$$\Pi_2 = \langle\rho\rangle\pi_2 . \tag{6}$$

Thus, the total number of independent piezoresistance coefficients required to describe the piezoresistance in isotropic polycrystalline semiconductors is found to be two, i.e., π_1 and π_2.

Any other quantities appearing in eqs. (1) to (4) can be contracted using *Voigt* notation[10] (see Appendix A3). Using the conventions defined above and *Voigt* notation, eq. (4) can be expressed as follows in the case of a Hall-type four-terminal piezoresistor subjected to in-plane stresses.

In the case where the electric field vector $\langle E_i\rangle$ and the current density vector $\langle J_j\rangle$ are parallel to each other within the same plane (two terminals of the Hall-type piezoresistor can be used), i.e., $\langle E_1\rangle \| \langle J_1\rangle$:

$$\langle\Delta\rho_1/\rho\rangle = \langle\pi_{11}\rangle\langle\sigma_1\rangle + \langle\pi_{22}\rangle\langle\sigma_2\rangle = (\pi_1 + 2\pi_2)\langle\sigma_1\rangle + \pi_1\langle\sigma_2\rangle = \pi_l\langle\sigma_1\rangle + \pi_t\langle\sigma_2\rangle$$
$$\text{and } \langle\pi_{16}\rangle = 0 . \tag{7}$$

In the case where $\langle E_2\rangle \| \langle J_2\rangle$:

$$\langle\Delta\rho_2/\rho\rangle = \langle\pi_{21}\rangle\langle\sigma_1\rangle + \langle\pi_{22}\rangle\langle\sigma_2\rangle = \pi_1\langle\sigma_1\rangle + (\pi_1 + 2\pi_2)\langle\sigma_2\rangle = \pi_t\langle\sigma_1\rangle + \pi_l\langle\sigma_2\rangle$$
$$\text{and } \langle\pi_{26}\rangle = 0 . \tag{8}$$

In contrast, if the two directions are orthogonal to each other (four terminals of the Hall-type piezoresistor can be used), i.e., $\langle E_1\rangle \perp \langle J_2\rangle$:

$$\langle\Delta\rho_6/\rho\rangle = 2\langle\pi_{66}\rangle\langle\sigma_6\rangle = 2\pi_2\langle\sigma_6\rangle = \pi_s\langle\sigma_6\rangle$$
$$\text{and } \langle\pi_{61}\rangle = \langle\pi_{62}\rangle = 0 . \tag{9}$$

In eqs. (7) to (9), $\pi_l \equiv \pi_1 + 2\pi_2$, $\pi_t \equiv \pi_1$ and $\pi_s \equiv 2\pi_2$ correspond to the conventional longitudinal, transverse and shear piezoresistance coefficients, respectively*. As can be seen from eqs. (7) to (9), the isotropic polycrystalline semiconductors are sensitive only to the in-plane normal stresses when the in-plane electric field vector and the in-plane current density vector are parallel to each other, and sensitive only to in-plane shear stress when the two in-plane vectors are orthogonal to each other.

* See ref. (13). The Bossche and Mollinger definitions of the isotropic poly-Si piezoresistance coefficients (π_l, π_t, π_p, π_{lp}, π_{tp}, π_{lt}) are related to the present paper by $\pi_l \equiv \pi_1+2\pi_2=\pi_l$, $\pi_t \equiv \pi_1=\pi_t$, $\pi_p \equiv \pi_1=\pi_t$, $\pi_{lp} \equiv 2\pi_2=\pi_s$, $\pi_{tp} \equiv 2\pi_2=\pi_s$ and $\pi_{lt} \equiv 2\pi_2=\pi_s$.

For the purpose of discussion in the following sections, we calculate the isotropic piezoresistance coefficients of p-type poly-Si. The piezoresistance coefficients of single-Si, with respect to the principal crystallographic frame, can be expressed as[16]

$$\pi_{ijkl} = \pi_{12}\delta_{ij}\delta_{kl} + \frac{\pi_{44}}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + (\pi_{11} - \pi_{12} - \pi_{44})\delta_{ijkl}, \tag{10}$$

where π_{11}, π_{12} and π_{44} are the fundamental piezoresistance coefficients.

Substituting eq. (10) into eq. (3) and taking into account eqs. (5) to (9), the two independent and conventional piezoresistance coefficients of p-type poly-Si can be expressed as

$$\pi_1 = \frac{1}{5}(\pi_{11} + 4\pi_{12} - \pi_{44}) \cong -\frac{1}{5}\pi_{44} \tag{11}$$

$$\pi_2 = \frac{1}{10}(2\pi_{11} - 2\pi_{12} + 3\pi_{44}) \cong \frac{3}{10}\pi_{44} \tag{12}$$

$$\pi_l \cong \frac{2}{5}\pi_{44} \tag{13}$$

$$\pi_t \cong -\frac{1}{5}\pi_{44} \tag{14}$$

$$\pi_s \cong \frac{3}{5}\pi_{44}, \tag{15}$$

where the resistivity of poly-Si is assumed to approach that of single-Si (this assumption is valid for the higher impurity range, *e.g.*, $>10^{19}$cm^{-3}[6,9,10,12]) and the relation $|\pi_{44}| \gg |\pi_{11}|, |\pi_{12}|$ for p-type single-Si is used.

The matrix forms of the isotropic piezoresistance coefficients for polycrystalline semiconductors $\langle \pi_{ijkl} \rangle$, and the anisotropic piezoresistance coefficients for cubic single- crystalline semiconductors π_{ijkl}, are

$$\begin{bmatrix} \pi_1 + 2\pi_2(\pi_{11}) & \pi_1(\pi_{12}) & \pi_1(\pi_{12}) & 0 & 0 & 0 \\ \pi_1(\pi_{12}) & \pi_1 + 2\pi_2(\pi_{11}) & \pi_1(\pi_{12}) & 0 & 0 & 0 \\ \pi_1(\pi_{12}) & \pi_1(\pi_{12}) & \pi_1 + 2\pi_2(\pi_{11}) & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_2(\pi_{44}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_2(\pi_{44}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_2(\pi_{44}) \end{bmatrix}, \tag{16}$$

where the single-crystalline components are in parentheses ( ).

4. Fabrication

Figure 1 shows the fabrication process for a p-type poly-Si piezoresistor. The starting material consists of an n-type Si (100) substrate with a SiO_2 isolation layer (0.7 μm-thick) and a p-type poly-Si layer (0.5 μm-thick) on which it was prepared. The impurity concentration N of poly-Si was controlled by the implantation of boron ions. The values $N = 10^{19}$ cm^{-3} (sheet resistance = 1514 Ω/□), $N = 5\times10^{19}$ cm^{-3} (sheet resistance = 141.7 Ω/□) and $N = 10^{20}$ cm^{-3} (sheet resistance = 76.7 Ω/□) were adopted. The p-type poly-Si piezoresistors, aluminum wires and contact pads were delineated by photolithography and etching. Finally, sintering was performed in dry N_2 for 30 min at 450°C.

Figure 2 shows a schematic top-view of piezoresistors. Four kinds of arrangements, (A)-(a), (A)-(b), (B) and (C), were prepared to measure the isotropic piezoresistance of *p*-type poly-Si.

Figure 3 shows a typical example of piezoresistors fabricated on Si substrates. A rectangular beam (length = 22.5 mm, width = 3 mm, thickness = 0.545 mm) was cut from a Si substrate and used to apply stress to the piezoresistors via a simple cantilever bending system.[17]

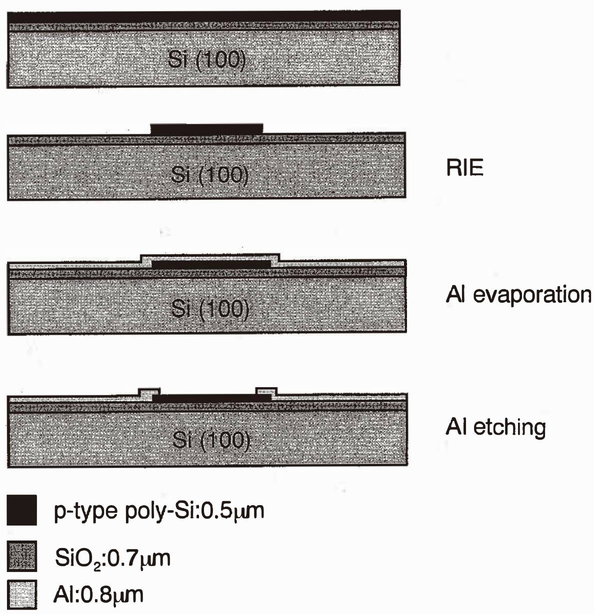

Fig. 1. Fabrication process of p-type poly-Si piezoresistor.

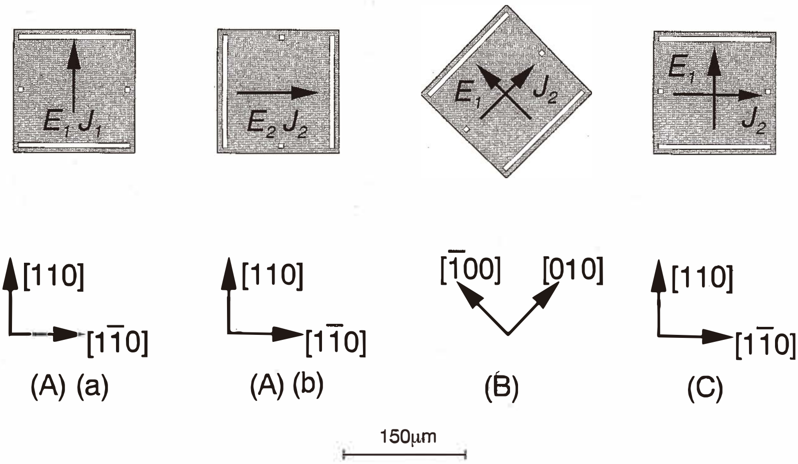

Fig. 2. Schematic top-view of fabricated piezoresistors.

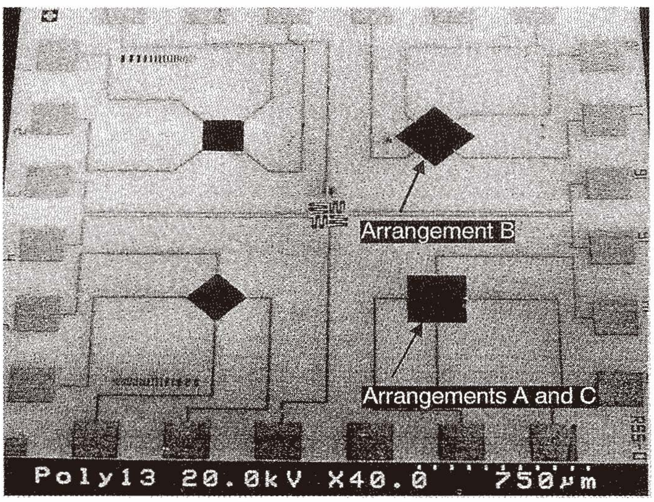

Fig. 3. Fabricated p-type poly-Si piezoresistors.

5. Experimental Procedure

5.1 *Stress state in poly-Si piezoresistor on Si cantilever*

Figure 4 shows the stress states on the Si cantilever surface and poly-Si piezoresistors on Si substrates under bending stress. The superscripts p and s indicate the quantities

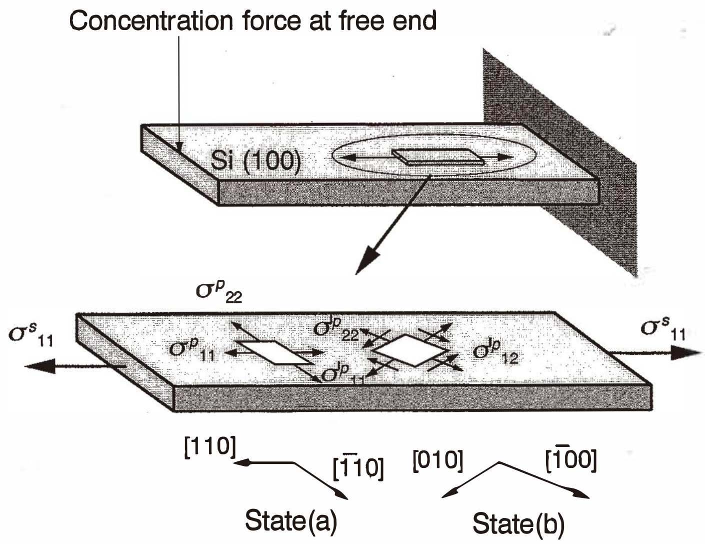

Fig. 4. Stress states of poly-Si piezoresistors on Si cantilever subjected to bending.

concerning the piezoresistor and Si substrate, respectively. Therefore, in the figure, σ^{p}_{ij} and σ'^{p}_{ij} represent the applied stresses in square elements of piezoresistors which are either (a) parallel (biaxial normal stresses) or (b) inclined 45 degrees to the bending stress axis (superposition of equiaxial normal stresses and shear stress), respectively. σ^{s}_{ij} and σ'^{s}_{ij} represent stresses on the Si substrate similar in manner to those on piezoresistors. Stresses σ^{p}_{ij} and σ'^{p}_{ij} in piezoresistors can be calculated using the elementary elasticity theory, under the assumption that strains on the Si substrate are completely transmitted to the piezoresistors:(17)

$$\sigma^{p}_{ij} = c^{p}_{ijkl} s^{s}_{klmn} \sigma^{s}_{mn} \tag{17}$$

$$\sigma'^{p}_{ij} = \sigma^{p}_{mn} l_{mi} l_{nj}, \tag{18}$$

where c^{s}_{ijkl} and s^{s}_{klmn} are the elastic constants of poly-Si and Si, respectively, and l_{mi} is the direction cosine between the bending stress axis and the axis which is inclined at an arbitrary angle against the bending stress axis.

c^{p}_{ijkl} and s^{s}_{klmn} are given as(17)

$$C^{p}_{ijkl} = E\nu/[(1+\nu)(1-2\nu)]\delta_{ij}\delta_{kl} + E/[2(1+2\nu)]\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) \tag{19}$$

$$S^{s}_{ijkl} = S_{12}\delta_{ij}\delta_{kl} + S_{44}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) + \left(S_{11} - S_{12} - 2S_{44}\right)\delta_{ijkl}. \tag{20}$$

Table 1 shows the mechanical properties which are used in the calculations.[18] E and ν are Young's modulus and Poisson's ratio of the poly-Si. S_{11}, S_{12} and S_{44} are cubic compliance constants of the single-Si. Note that the piezoresistor is subjected to biaxial stresses under uniaxial bending stress of the Si substrate (see Fig. 4). This is due to the difference in elastic constants between the piezoresistor and the Si substrate. In the following sections, we also use the *Voigt* notation to express the stress components (see Appendix A3).

5.2 *Experimental setup*

The individual cantilever specimens A, B and C have various configurations of piezoresistors, as shown in Fig. 5. Arrangement A has piezoresistors in which both $\langle E_1 \rangle$

Table 1
Elastic properties of poly-Si and Si.

Poly-Si		Si		
Young's modulus E (GPa)	Poisson's ratio ν	Elastic constant S_{11} (1/MPa)	Elastic constant S_{12} (1/MPa)	Elastic constant S_{44} (1/MPa)
160	0.25	7.68×10^{-6}	-2.14×10^{-6}	3.14×10^{-6}

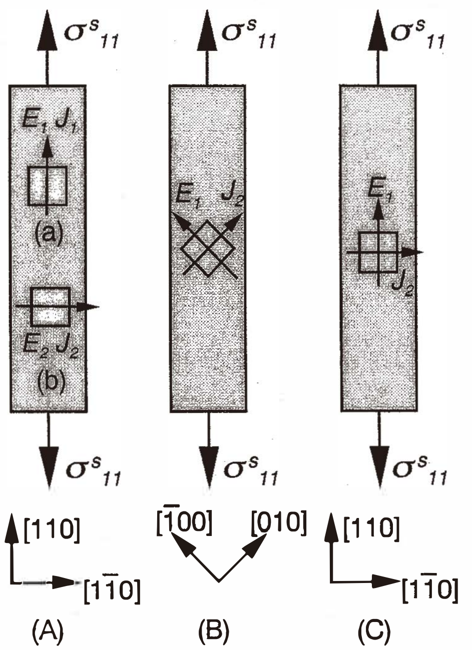

Fig. 5. Arrangements of poly-Si piezoresistors.

and $\langle J_1\rangle$, $\langle E_2\rangle$ and $\langle J_2\rangle$ are either parallel or perpendicular to the bending stress axis. The stress conditions of these piezoresistors correspond to state (a) in Fig. 4. Arrangement B has piezoresistors in which $\langle E_1\rangle$ and $\langle J_2\rangle$ are orthogonal to each other and inclined 45 degrees to the bending stress axis. The stress condition of these piezoresistors corresponds to state (b) in Fig. 4. Arrangement C has piezoresistors in which $\langle E_2\rangle$ is parallel and $\langle J_2\rangle$ is perpendicular to the bending stress axis. The stress condition of these piezoresistors corresponds to state (a) in Fig. 4.

Under the assumption that the geometrical effect on piezoresistance due to deformation can be neglected, the directly observed quantities in each arrangement are expressed by eqs. (7) to (9).

For arrangement A (a) (eq. (7)):

$$\Delta R / R = \langle \pi_{11} \rangle \sigma^{P}{}_{1} + \langle \pi_{12} \rangle \sigma^{P}{}_{2} = \pi_l \sigma^{P}{}_{1} + \pi_t \sigma^{P}{}_{2}. \tag{21}$$

For arrangement A (b) (eq. (8)):

$$\Delta R / R = \langle \pi_{11} \rangle \sigma^{P}{}_{2} + \langle \pi_{12} \rangle \sigma^{P}{}_{1} = \pi_l \sigma^{P}{}_{2} + \pi_t \sigma^{P}{}_{1}. \tag{22}$$

For arrangement B (eq. (9)):

$$\Delta V / V_{in} = 2\langle \pi_{66} \rangle \sigma'^{P}{}_{6} = \pi_s \sigma'^{P}{}_{6}. \tag{23}$$

For arrangement C (eq. (9)):

$$\Delta V / V_{in} = \langle \pi_{61} \rangle \sigma^{P}{}_{1} + \langle \pi_{62} \rangle \sigma^{P}{}_{2} = 0. \tag{24}$$

Here, $\Delta R/R$ is the change in resistance between two terminals, ΔV is the change in transverse voltage between two terminals due to the applied stresses and V_{in} is the applied voltage between the other two terminals of the Hall-type four-terminal piezoresistor.

The coefficients π_l, π_t and π_s can be determined by the combination of eqs. (21), (22), and eq. (23), respectively. Equations (21), (22) and (24) can be used to confirm the in-plane piezoresistance property when the in-plane electric field vector and the in-plane current density vector are parallel to each other ($\langle E_1\rangle \| \langle J_1\rangle$ and $\langle E_2\rangle \| \langle J_2\rangle$). In contrast, eqs. (23) and (24) can be used to confirm the in-plane piezoresistance property when they are orthogonal to each other ($\langle E_1\rangle \perp \langle J_2\rangle$).

6. Experimental Results

Figure 6 shows $\Delta R/R$ against the applied bending stress $\sigma^{s}{}_{1}$ on the Si substrate at the position of the piezoresistors in A(a) and A(b). Figure 7 shows $\Delta V/V_{in}$ against the applied

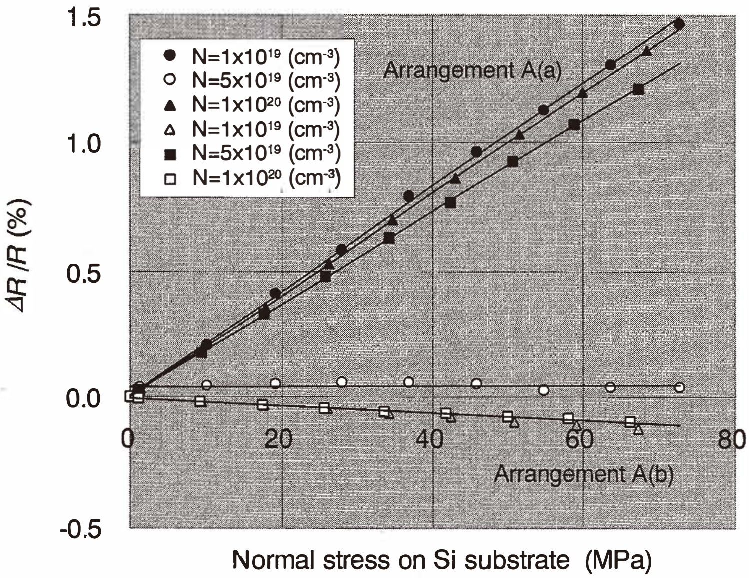

Fig. 6. Experimental results for arrangements A(a) and A(b).

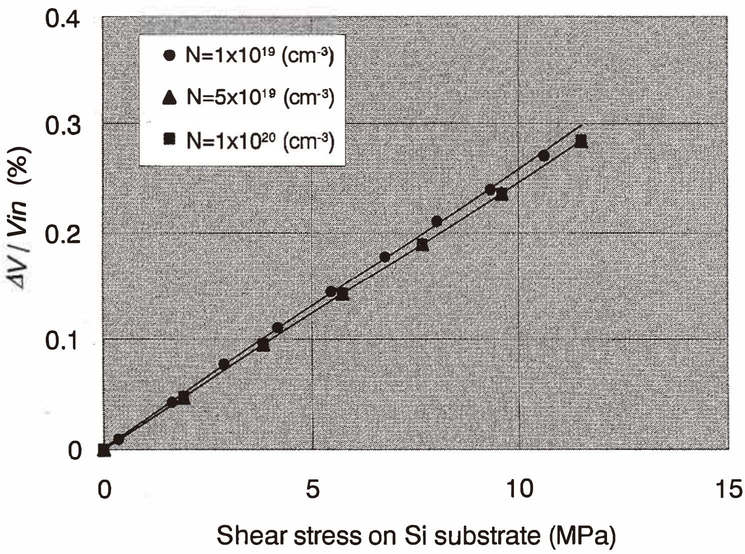

Fig. 7. Experimental result for arrangement B.

shear stress σ'^s_6 on the Si substrate at the position of the piezoresistor in B. Figure 8 shows $\Delta V/V_{in}$ against the applied bending stress σ^s_1 on the Si substrate at the position of the piezoresistor in C. Table 2 shows the piezoresistance coefficients of p-type poly-Si, π_l, π_t and π_s, which were determined by the combination of eqs. (21) and (22), and eq. (24).

7. Discussion

The experimental results of arrangements B and C reveal that $2\langle\pi_{66}\rangle = 2\pi_2 = \pi_s$ has a positive value and that $\langle\pi_{61}\rangle = \langle\pi_{62}\rangle = 0$, respectively. The results completely satisfy eq. (9). Therefore, it can be concluded that p-type poly-Si is sensitive only to in-plane shear stress when $\langle E_1\rangle \perp \langle J_2\rangle$. In contrast, the experimental results of arrangements A(a) and A(b) reveal that $\langle\pi_{11}\rangle = \pi_l$ is positive and that $\langle\pi_{12}\rangle = \pi_t$ is negative, respectively. The isotropic piezoresistance coefficient $\langle\pi_{\alpha\beta}\rangle$ is symmetrical with respect to the index, as can be seen from eq. (3), i.e., $\langle\pi_{\alpha\beta}\rangle = \langle\pi_{\beta\alpha}\rangle$. As a consequence, $\langle\pi_{61}\rangle = \langle\pi_{62}\rangle = 0$, which is obtained from arrangement C, also satisfies $\langle\pi_{16}\rangle = \langle\pi_{26}\rangle = 0$. Therefore, it can be concluded that p-type poly-Si satisfies eqs. (7) and (8) and is sensitive only to in-plane normal stresses when $\langle E_1\rangle \| \langle J_1\rangle$ and $\langle E_2\rangle \| \langle J_2\rangle$. The results obtained are also applicable to isotropic polycrystalline semiconductors with cubic single crystals, i.e., β-SiC and diamond.

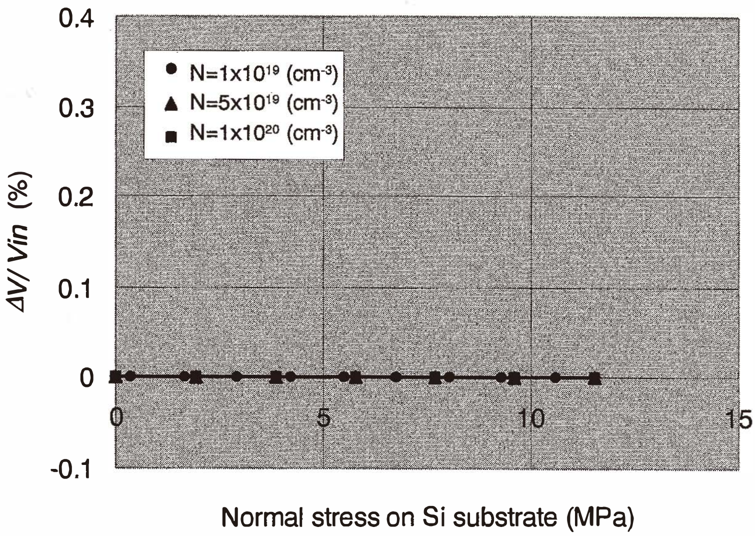

Fig. 8. Experimental result for arrangement C.

In Table 2, the experimental values of π_l for each impurity concentration are normalized with π_l, which are calculated from the experimental values π_t and π_s, within 10% error. Results of the above-mentioned in-plane piezoresistance properties and the self-consistent calculation of π_l support the theoretical prediction, i.e., the number of independent coefficients of the isotropic poly-Si is two.

The piezoresistors used in the present work cannot determine the sensitivities for out-of-plane normal and shear stresses. From the results for the in-plane piezoresistance properties and the two independent piezoresistance coefficients, it is deduced that the magnitude of the sensitivities for out-of-plane stresses is same as that for the in-plane stresses. As noted by Bessche and Mollinger,[13] the out-of-plane stress sensitivities are important for solving the molded IC reliability problems.

Table 2 also shows experimental and theoretical values of π_l, π_t and π_s, respectively. The theoretical values were obtained by substituting π_{44} of p-type single-Si into eqs. (13) to (15). The values of π_{44} were taken from the literature of Kerr and Milnes.[18] Discrepancies between the experimental and theoretical values of π_l and π_s are relatively large in the case where impurity concentrations are $N = 10^{19}$ cm^{-3} and $N = 5\times10^{19}$ cm^{-3}. Discrepancies become smaller at $N = 10^{20}$ cm^{-3}, and experimental and theoretical values of π_l and π_s coincide within 25% error, while discrepancy between the experimental and theoretical values of π_t are very large at all impurity concentrations. The anomalous behavior of π_t was noted by several researchers and the reason cannot be clarified to date.[8,11] However, it is concluded that direct prediction of the piezoresistance coefficients of poly-Si from that of the single-Si is in poor agreement with the experiments except in terms of sign and order of magnitude.

It is recognized that grain boundary piezoresistance of the poly-Si has an important role in addition to grain piezoresistance.[9,10,12] One possible way to interpret the grain boundary piezoresistance of the poly-Si was proposed by French and Evans.[9,10] They used the Schottky-type barrier model to determine the grain boundary piezoresistance coefficients. However, the carrier trapping model for carrier transport through the grain boundary of poly-Si, proposed by Kamins,[21] has considerable support due to consistency with experiments.[22–24] Thus, the grain boundary piezoresistance of the poly-Si is discussed based on the carrier trapping model. Electrical conductivity σ_g of the grain boundary of p-type-Si is expressed as

Table 2
Piezoresistance coefficients of p-type poly-Si.

N	$\pi_l \equiv \pi_1 + 2\,\pi_2$ ($\times 10^{-5}$ 1/MPa)		$\pi_t \equiv \pi_1$ ($\times 10^{-5}$ 1/MPa)		$\pi_s \equiv 2\,\pi_2$ ($\times 10^{-5}$ 1/MPa)		$\pi_t + \pi_s \equiv \pi_1 + 2\,\pi_2$ ($\times 10^{-5}$ 1/MPa)
(cm^{-3})	Exp.	Eq. (13)	Exp.	Eq. (14)	Exp.	Eq. (15)	Exp.
1×10^{19}	21.0	33.0	−3.5	−16.5	26.3	49.5	22.8
5×10^{19}	21.2	27.0	−5.8	−13.5	26.9	40.5	21.1
1×10^{20}	19.1	24.0	−5.1	−12.0	26.5	36.0	21.4

$$\sigma_g = \frac{q^2 W_B}{\sqrt{2\pi}(kT)^{3/2}} \exp\left(-\frac{qV_B}{kT}\right)\left(\frac{P_1}{\sqrt{m_1}} + \frac{P_2}{\sqrt{m_2}}\right), \tag{25}$$

where q is electric charge, k is the Boltzmann constant, T is absolute temperature, W_B is the length of the depletion region at the grain boundary, V_B is the potential barrier height at grain boundary, and p_i and m_i are hole concentration and effective mass for heavy hole (i=1) and light hole (i=2), respectively. Change in σ_g due to uniaxial stress Σ is defined as $\Delta\sigma_g$. By differentiating eq. (25) and assuming that V_B is unchanged after deformation, the longitudinal and transverse grain boundary piezoresistance coefficients, π_{gl} and π_{gt}, can be obtained as

$$\pi_{gl,t} = -\frac{\Delta\sigma_{gl,t}}{\sigma_{gl,t}\Sigma_{l,t}} = -\frac{\dfrac{\Delta P_{1l,t}}{\sqrt{m_{1l,t}}} + \dfrac{\Delta P_{2l,t}}{\sqrt{m_{2l,t}}} + P_{2l,t}\Delta\left(\dfrac{1}{\sqrt{m_{2l,t}}}\right)}{\dfrac{P_{1l,t}}{\sqrt{m_{1l,t}}} + \dfrac{P_{2l,t}}{\sqrt{m_{2l,t}}}} \tag{26}$$

The physical meaning of eq. (26) is that the grain boundary piezoresistance is caused by hole transfer between two holes and light hole mass shift due to stress.(25) These phenomena for p-type single-Si have anisotropic features and are briefly discussed by Hensel and Feher.(25) Introducing an assumption that the hole transfer and the light hole mass shift at the grain boundary have isotropic properties, and the grain boundaries behave like randomly oriented single-Si grains, π_{gl} is found to be 6.6×10^{-5} 1/MPa at $N = 10^{18}$ cm^{-3} and 3.3×10^{-5} 1/MPa at $N = 10^{19}$ cm^{-3}, while π_{gt} is found to be -3.3×10^{-5} 1/MPa at $N = 10^{18}$ cm^{-3} and -1.7×10^{-5} 1/MPa at $N = 10^{19}$ cm^{-3} at room temperature. In the case of isotropy, shear grain boundary piezoresistance coefficient π_{gs} can be deduced from eqs. (7) to (9) as $\pi_{gs} = \pi_{gl} - \pi_{gt}$. Then, π_{gs} is found to be 9.9×10^{-5} 1/MPa at $N = 10^{18}$ cm^{-3} and 5.0×10^{-5} 1/MPa at $N = 10^{19}$ cm^{-3} at room temperature. In the calculations, averaging for the hole transfer and the light hole mass shift of single-Si grains with respect to all crystallographic orientations is adopted, and values are taken from the study by Hensel and Feher.(25) From the above results, the grain boundary piezoresistance appears to be significant in the lower impurity range, i.e., $N < 10^{18}$ cm^{-3}, and insignificant in the case of $N > 10^{19}$ cm^{-3}. Furthermore, the lower impurity range is not suitable for sensor fabrication due to lower sensitivity and lack of temperature characteristics.(7,9,10,12)

As a comparison, the piezoresistance coefficients π_l, π_t and π_s of p-type poly-Si obtained by French and Evans are reviewed here.(9,10) They determined longitudinal, transverse and shear gauge factors, G_l, G_t, and G_s, which correspond to π_l, π_t and π_s, respectively. Therefore, as suggested by French and Evans,(9,10) π_l, π_t and π_s are calculated by relations $G_l = E\pi_l$, $G_t = E\pi_t$, and $G_s = E\pi_s/(2(1+\nu))$, respectively. The values of E and ν are taken from Table 1. Then, $\pi_l = 23.8 \times 10^{-5}$ 1/MPa, $\pi_t = -10.0 \times 10^{-5}$ 1/MPa and $\pi_s = 19.0 \times 10^{-5}$ 1/MPa at $N = 10^{19}$ cm^{-3}, while $\pi_l = 18.8 \times 10^{-5}$ 1/MPa, $\pi_t = -7.0 \times 10^{-5}$ 1/MPa and $\pi_s = 14.0 \times 10^{-5}$ 1/MPa at $N = 10^{20}$ cm^{-3} are obtained. These values are on the same orders of magnitude as the present work.

8. Conclusions

An isotropic piezoresistance equation for the poly-Si is derived. Based on the derived equation, the total number of independent piezoresistance coefficients needed to describe the isotropic piezoresistance effect of poly-Si is found to be two. It is also derived that the change in resistance is only induced by normal stresses while the change in transverse voltage is only induced by shear stresses. The obtained results are also applicable to isotropic polycrystalline semiconductors with cubic single crystals, i.e., β-SiC and diamond, as well as poly-Si. Experiments on the isotropic p-type poly-Si Hall-type piezoresistors were carried out under in-plane normal and in-plane shear stress conditions and the above-mentioned piezoresistance properties were confirmed. The relationship between the two independent piezoresistance coefficients of the isotropic poly-Si and the three independent piezoresistance coefficients of single-Si was derived using a crystallographic orientation averaging (overall averaging) method. However, it is concluded that direct prediction of the piezoresistance coefficients of isotropic poly-Si from that of the single-Si is in poor agreement with the experimental values except in terms of sign and order of magnitude.

References

1 W. G. Pfann and R. N. Thurston: J. Applied Physics **32** (1961) 2008.
2 Y. Kanda and A. Yasukawa: Sensors and Actuators **2** (1982) 283.
3 Y. Kanda: Sensors and Actuators **4** (1983) 199.
4 M. Bao and Y. Wang: Sensors and Actuators **12** (1987) 49.
5 Y. Kanda: Japanese J. Applied Physics **26** (1987) 1031.
6 J. Y. W. Seto: J. Applied Physics **47** (1976) 4780.
7 W. Germer and W. Todt: Sensors and Actuators **4** (1983) 183.
8 O. Dossel: Sensors and Actuators **6** (1984) 169.
9 P. J. French and A. G. R. Evans: Sensors and Actuators **8** (1985) 219.
10 P. J. French and A. G. R. Evans: Solid-State Electronics **32** (1989) 1.
11 E. Obermeier and P. Kopystynski: Sensors and Actuators **30** (1992) 149.
12 P. J. French and A. G. R. Evans: Sensors and Actuators **15** (1988) 257.
13 A. Bossche and J. R. Mollinger: Sensors and Actuators A **62** (1997) 475.
14 W. P. Mason and R. N. Thurston: J. Acoustical Society of America **29** (1957) 1096.
15 *e.g.*, T. Mura: Micromechanics of Defects in Solids, 2nd. ed. (Kluwer Academic Publishers, Amsterdam, 1987).
16 T. Toriyama and S. Sugiyama: IEEE Proceedings of the 1998 International Symposium on Micromechatronics and Human Science (IEEE, Nagoya, 1998) p.183.
17 *e.g.*, I. H. Shames: Introduction to Solid Mechanics, 2nd. ed. (Prentice-Hall Inc, New York, 1989).
18 O. Tabata, K. Kawahata, S. Sugiyama and I. Igarashi: Sensors and Actuators **20** (1989) 135.
19 *e.g.*, Y. C. Fung: First course in continuum mechanics, 3rd. ed. (Prentice-Hall Inc, New York, 1994) 170.
20 D. R. Keer and A. G. Milnes: J. Applied Physics **34** (1963) 727.
21 T. I. Kamins: J. Applied Physics **42** (1971) 4357.
22 J. Y. W. Seto: J. Applied Physics **46** (1975) 5247.
23 N. C. C. Lu, L. Gerzberg, C. Y. Lu and J. D. Meindl: IEEE Transactions on Electron Devices ED-**28** (1981) 818.

24 D. P. Joshi and R.S.Srivastava: IEEE Transactions on Electron Devices ED-**31** (1984) 920.
25 J. C. Hensel and G. Feher: Physical Review **129** (1963) 1041.

Appendices

A1. *Overall averaging method*

We define texture in polycrystalline semiconductors by means of the crystalline orientation distribution function $\omega(\Omega)$. The probability of grains having values between $\omega(\Omega) \equiv \omega(\psi, \theta, \phi)$ and $\omega(\Omega + d\Omega) \equiv \omega(\psi + d\psi, \theta + d\theta, \phi + d\phi)$ can be given as $\omega(\Omega)d\Omega \equiv \omega(\psi, \theta, \phi)\sin\theta d\theta d\psi d\phi$, where ψ, θ and ϕ are the Euler angles.[15] The Euler angles specify arbitrary crystallographic orientations with respect to the principal crystallographic orientation. The averaging of $\omega(\Omega) \equiv \omega(\psi, \theta, \phi)$ with respect to all crystalline orientations can be expressed as

$$\int_{\Omega} \omega(\Omega)d\Omega = \int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi}\omega(\psi,\theta,\phi)\sin\theta d\theta d\psi d\phi = 1. \tag{A1}$$

In case of the isotropy, $\omega(\Omega) = \dfrac{1}{8\pi^2}$.

The single-crystalline piezoresistance tensor $\pi'_{ijkl}(\Omega)$ having orientation $\Omega \equiv (\psi,\theta,\phi)$ can be described as

$$\pi'_{ijkl}(\Omega) = \pi_{mnpq} l_{mi} l_{nj} l_{pk} l_{ql}, \tag{A2}$$

where l_{mi} is the direction cosine between an arbitrary orientation and the crystallographic axes.

Therefore, averaging of the single-crystalline tensor $\pi'_{ijkl}(\Omega)$ with respect to the crystallographic orientations can be expressed as

$$\langle\pi_{ijkl}\rangle = \int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi}\pi'_{ijkl}\omega(\psi,\theta,\phi)\sin\theta d\theta d\psi d\phi = \pi_{mnpq}\int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi} l_{mi} l_{nj} l_{pk} l_{ql}\omega(\psi,\theta,\phi)\sin\theta d\theta d\psi d\phi, \tag{A3}$$

where $\langle\pi_{ijkl}\rangle$ is the desired polycrystalline piezoresistance tensor.

A2. *Note on orientation averaging in case of isotropy*

Here, a derivation procedure of eq. (3) appearing in the text is described. In the case of complete random distribution of single crystals in a polycrystalline semiconductor, the polycrystalline tensor $\langle\pi_{ijkl}\rangle$ has an isotropic property. In this case, $\langle\pi_{ijkl}\rangle$ can be derived using the isotropic tensor property in place of direct and tedious calculation based on eq. (A3).

In general, a four rank isotropic tensor with the symmetric property, $T_{ijkl} = T_{jikl} = T_{ijlk}$, can be expressed as[19]

$$T_{ijkl} = T_1\delta_{ij}\delta_{kl} + T_2(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}), \tag{A4}$$

where

$$\begin{aligned} T_1 &= \frac{1}{15}\left(2T_{ppqq} - T_{pqpq}\right) \\ T_2 &= \frac{1}{30}\left(3T_{pqqq} - T_{ppqq}\right). \end{aligned} \tag{A5}$$

The polycrystalline tensor $\langle \pi_{ijkl} \rangle$ has the above-mentioned symmetric property. Averaging of the single-crystalline tensor π_{ijkl} with respect to all Euler angles derives $\langle \pi_{ijkl} \rangle$ with the isotropic tensor property. Thus, $\langle \pi_{ijkl} \rangle$ can be expressed using eqs. (A4) and (A5) as

$$\langle \pi_{ijkl} \rangle = \Pi_1\delta_{ij}\delta_{kl} + \Pi_2(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}), \tag{A6}$$

where

$$\begin{aligned} \Pi_1 &= \frac{1}{15}\left(2\langle \pi_{ppqq} \rangle - \langle \pi_{pqpq} \rangle\right) \\ \Pi_2 &= \frac{1}{30}\left(3\langle \pi_{pqpq} \rangle - \langle \pi_{ppqq} \rangle\right). \end{aligned} \tag{A7}$$

In eq. (A7), $\langle \pi_{pqpq} \rangle$ and $\langle \pi_{ppqq} \rangle$ are the scalar invariants under coordinate transformations. Therefore, $\langle \pi_{pqpq} \rangle = \pi_{pqpq}$ and $\langle \pi_{ppqq} \rangle = \pi_{ppqq}$ can be identified. The proof of $\langle \pi_{pqpq} \rangle = \pi_{pqpq}$ and $\langle \pi_{ppqq} \rangle = \pi_{ppqq}$ can be obtained with the aid of eq. (A3). The results are

$$\begin{aligned} \langle \pi_{iikk} \rangle &= \pi_{mnpq} \frac{1}{8\pi^2} \int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi} l_{mi} l_{ni} l_{pk} l_{qk} \sin\theta d\theta d\psi d\phi \\ &= \pi_{mnpq} \frac{1}{8\pi^2} \int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi} \delta_{mn}\delta_{pq} \sin\theta d\theta d\psi d\phi \\ &= \pi_{mmpp} \frac{1}{8\pi^2} 8\pi^2 = \pi_{mmpp}, \end{aligned} \tag{A8}$$

$$\begin{aligned} \langle \pi_{ikik} \rangle &= \pi_{mnpq} \frac{1}{8\pi^2} \int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi} l_{mi} l_{nk} l_{pi} l_{qk} \sin\theta d\theta d\psi d\phi \\ &= \pi_{mnpq} \frac{1}{8\pi^2} \int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi} \delta_{mp}\delta_{nq} \sin\theta d\theta d\psi d\phi \\ &= \pi_{pqpq} \frac{1}{8\pi^2} 8\pi^2 = \pi_{pqpq}, \end{aligned} \tag{A9}$$

where

$$\int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi}\sin\theta d\theta d\psi d\phi = 8\pi^2 \tag{A10}$$

can be used.

Finally, eq. (A7) can be rewritten as:

$$\begin{aligned} \Pi_1 &= \frac{1}{15}\left(2\pi_{ppqq} - \pi_{pqpq}\right) \\ \Pi_2 &= \frac{1}{30}\left(3\pi_{pqpq} - \pi_{ppqq}\right). \end{aligned} \tag{A11}$$

It is shown that the set of eqs. (A6) and (A11) correspond to eq. (3) in the text.

A3. *Note on Voigt notation*

The index pairs (11), (22), (33), (23) = (32), (31) = (13), and (12) = (21) of a four rank tensor and a two rank tensor can be contracted by single indices 1, 2, 3, 4, 5 and 6, respectively.

For the piezoresistance tensor:(14)

$$\begin{aligned} &\pi_{ijkl} \equiv \pi_{\alpha\beta}\ (\alpha = 1, 2, 3, 4, 5, 6\text{: } \beta = 1, 2, 3) \\ &2\pi_{ijkl} \equiv \pi_{\alpha\beta}\ (\alpha = 1, 2, 3, 4, 5, 6\text{: } \beta = 4, 5, 6). \end{aligned}$$

For the elastic stiffness and compliance tensors:(15)

$$\begin{aligned} &C_{ijkl} \equiv C_{\alpha\beta}\ (\alpha = 1, 2, 3, 4, 5, 6\text{: } \beta = 1, 2, 3, 4, 5, 6) \\ &S_{ijkl} \equiv S_{\alpha\beta}\ (\alpha = 1, 2, 3, 4, 5, 6\text{: } \beta = 1, 2, 3, 4, 5, 6). \end{aligned}$$

For the resistivity tensor:(14)

$$\rho_{ij} \equiv \rho_{\alpha}\ (\alpha = 1, 2, 3, 4, 5, 6).$$

For the stress tensor:(15)

$$\sigma_{ij} \equiv \sigma_{\alpha}\ (\alpha = 1, 2, 3, 4, 5, 6).$$