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Circuit Implementation of Chaos Synchronization Based on Advanced Ameliorated Dynamic Control

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In this paper, Lü-like chaotic systems, which failed to achieve chaos synchronization via a general dynamic control scheme, were chosen for the circuit implementation of chaos synchronization based on our proposed advanced ameliorated dynamic control, which combined two controllers involving different variables into one controller. This not only simplified the controller but also reduced the number of electronic components. Experimental results show that our proposed advanced ameliorated dynamic control is feasible and effective and can be applied to chaos synchronization sensors.

1. Introduction

Chaos synchronization⁽¹⁻⁵⁾ is an important topic in nonlinear science and has been developed extensively. Various schemes of chaos synchronization have been proposed, including linear and nonlinear feedback control,⁽⁶⁻⁸⁾ adaptive control,⁽⁹⁻¹¹⁾ and active control.⁽¹²⁻¹⁴⁾ Chaos synchronization has been used to develop new methods, control more complex systems, and achieve synchronization more efficiently. Moreover, the applications of this concept have been explored in a wide range of fields, such as secure communication^(15,16) and electronic circuit design.^(17,18)

In 1998, static feedback control, which successfully synchronized many systems, was proposed. However, this approach failed as systems became increasingly complicated. To overcome this problem, Ramirez *et al.*⁽¹⁹⁾ proposed a scheme in which the static controller was replaced with a dynamic controller, and many complex systems including ones that could not be synchronized previously were successfully synchronized. The dynamic controller evolved by a differential equation was capable of updating the self-estimated value to give the error function asymptotic stability. Nonetheless, we doubt that this scheme based on dynamic control will work for all complex systems.

*Corresponding author: e-mail: tea0075@gmail.com https://doi.org/10.18494/SAM4052 We re-examined this scheme based on dynamic control and found that it failed to synchronize Lü-like chaotic systems.^(20,21) Then, we ameliorated the dynamic controller, that is, the dynamic controller was replaced with two dynamic ones, each evolved by an individual first-order differential equation, to enhance the coupling between systems. In 2022, Lu *et al.*⁽²²⁾ proposed a scheme based on ameliorated dynamic control and successfully achieved the chaos synchronization of Lü-like chaotic systems. In this study, we further propose a scheme based on advanced ameliorated dynamic control, that is, two controllers based on first-order differential equations are combined into one controller based on a first-order differential equation, and we implement the circuit. This scheme not only simplifies the controller system, but also reduces the number of electronic components. Lü-like chaotic systems are used to verify the validity of the implementation of our proposed scheme, which can be applied to sensor circuits.

The paper is organized as follows. First, Sect. 2 presents the synchronization scheme based on ameliorated dynamic control. The proposed scheme based on advanced ameliorated dynamic control is introduced in Sect. 3. In Sect. 4, the experimental results for Lü-like chaotic systems are presented. Finally, the discussion and conclusions are provided in Sects. 5 and 6, respectively.

2. Synchronization Scheme Based on Ameliorated Dynamic Control

This scheme was first proposed in 2022. Consider the following master-slave systems:

Master:
$$\begin{cases} \dot{x}_m = F(x_m), \\ y_m = Cx_m, \end{cases}$$
(1)

Slave:
$$\begin{cases} \dot{x}_s = F(x_s) - B[h_1 \quad h_2], \\ y_s = Cx_s, \end{cases}$$
 (2)

Dynamic controller:
$$\left\{ \begin{bmatrix} \dot{h} \\ \dot{h} \end{bmatrix} = -\alpha \begin{bmatrix} h \\ h \end{bmatrix} - kHC(x_m - x_s). \right\}$$
 (3)

Here, $x_m, x_s \in \mathbb{R}^n$ are the state vectors of the master and slave systems, respectively, $y_i \in \mathbb{R}, i = m, s$ are the corresponding outputs, function *F* is assumed to be sufficiently smooth, $B \in \mathbb{R}^{n \times 2}$, $C \in \mathbb{R}^{n \times n}$, and $H \in \mathbb{R}^{2 \times n}$ are constant vectors, $h_1 \in \mathbb{R}$ and $h_2 \in \mathbb{R}$ are the dynamic controllers, $k \in \mathbb{R}_+$ is the coupling strength, and $\alpha \in \mathbb{R}_+$ is a design parameter.

Assume that the nonlinear function *F* consists of linear and nonlinear parts:

$$F(x_i) = Ax_i + f(x_i), i = m, s,$$
(4)

where $A \in \mathbb{R}^{n \times n}$ is a constant matrix.

Then, the error dynamics for the systems in Eqs. (1)-(3) is obtained as

$$\begin{bmatrix} \dot{e} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} A & B \\ -kHC & -\alpha \end{bmatrix} \begin{bmatrix} e \\ h \end{bmatrix} + \begin{bmatrix} g(t,e) \\ 0 \end{bmatrix}, \tilde{e} = \begin{bmatrix} \dot{e} \\ \dot{h} \end{bmatrix}, \\ \overline{A} = \begin{bmatrix} A & B \\ -kHC & -\alpha \end{bmatrix} \begin{bmatrix} e \\ h \end{bmatrix}, \overline{g}(t,\tilde{e}) = \begin{bmatrix} g(t,e) \\ 0 \end{bmatrix},$$
(5)

where $e = x_m - x_s$, $h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$, $g(t, e) = f(x_m) - f(x_s)$, $\tilde{e} \in \mathbb{R}^{n+2}$ is the state vector, $h \in \mathbb{R}^2$, and matrix $\overline{A} \in \mathbb{R}^{(n+2)\times(n+2)}$ is assumed to be Hurwitz.⁽¹⁷⁾ Since the trajectories of the master system are bounded, the term $\bar{g}(t, \tilde{e})$ can be regarded as a perturbation that will vanish on *e* if it satisfies

$$\left\| \overline{g}\left(t, \tilde{e}\right) \right\|_{2} \leq \left\| \tilde{e} \right\|_{2}, \forall t \ge 0, \forall \tilde{e} \in D \subset \mathbb{R}^{n-2},$$
(6)

where $\|\cdot\|_2$ denotes the Euclidean norm. The stability properties of the error dynamics in Eq. (5) can be inspected as follows. First, consider the quadratic Lyapunov function

$$V(\tilde{e}) = \tilde{e}^T P \tilde{e}, \tag{7}$$

where $P \in \mathbb{R}^{(n+2)\times(n+2)}$ is a positive definite and symmetric matrix that is the solution of the Lyapunov equation

$$P\overline{A} + \overline{A}^T P = -Q. \tag{8}$$

Here, $Q \in \mathbb{R}^{(n+2)\times(n+2)}$ is a positive definite and symmetric matrix: a standard choice is Q = I, where I is the identity matrix of appropriate dimensions. In addition, a unique solution for Eq. (8), $P = P^r > 0$, always exists because \overline{A} in Eq. (5) has been assumed to be Hurwitz.

Next, through calculations, the time derivative of the Lyapunov function in Eq. (7) satisfies

$$\dot{V}(\tilde{e}) \leq -\left[\lambda_{\min}\left(Q\right) - 2\lambda_{\max}\left(P\right)\gamma\right] \|\tilde{e}\|_{2}^{2}, \qquad (9)$$

where $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ denote the minimum and maximum eigenvalues, respectively.

When \overline{A} is assumed to be Hurwitz, a sufficient condition for the local stability of the system in Eq. (5) is that the bound γ on the perturbation term in Eq. (6) is sufficiently small to satisfy

$$\gamma < \frac{\lambda_{\min}\left(Q\right)}{2\lambda_{\max}\left(P\right)}.$$
(10)

3. Synchronization Scheme Based on Advanced Ameliorated Dynamic Control

Consider the following master-slave systems:

Master:
$$\begin{cases} \dot{x}_m = F(x_m), \\ y_m = Cx_m, \end{cases}$$
(11)

Slave:
$$\begin{cases} \dot{x}_s = F(x_s) - Dh, \\ y_s = Cx_s, \end{cases}$$
 (12)

Dynamic controllers:
$$\{\dot{h} = -\alpha h - kJC(x_m - x_s)\}$$
. (13)

Here, $x_m, x_s \in \mathbb{R}^n$ are the state vectors of the master and slave systems, respectively, $y_i \in \mathbb{R}, i = m, s$ are the corresponding outputs, function *F* is assumed to be sufficiently smooth, $C \in \mathbb{R}^{n \times n}$, $D \in \mathbb{R}^{n \times 1}$, and $J \in \mathbb{R}^{1 \times n}$ are constant vectors, *h* is the dynamic controller, α is a design parameter, and *k* is the coupling strength.

Since the underlying theory is the same and the calculation is similar to that in Sect. 2, except that the dimension of the error dynamics is decreased, the following process is omitted. The predominant feature of our proposed synchronization scheme based on advanced ameliorated control is that two controllers are combined into one to simplify the controller system that facilitates the realization of the circuit.

4. Experimental Results

4.1 Numerical simulation

Take Lü-like chaotic^(20,21) systems as an example.

Master:
$$\begin{cases} \dot{x}_m = a(y_m - x_m) + dx_m z_m \\ \dot{y}_m = fy_m - x_m z_m \\ \dot{z}_m = cz_m + x_m y_m - ex_m^2 \end{cases}$$
(14)

Slave:
$$\begin{cases} \dot{x}_{s} = a(y_{s} - x_{s}) + dx_{s}z_{s} - h \\ \dot{y}_{s} = fy_{s} - x_{s}z_{s} - h \\ \dot{z}_{s} = cz_{s} + x_{s}y_{s} - ex_{s}^{2} \end{cases}$$
(15)

Dynamic controllers:
$$\dot{h} = -\alpha h - k \lfloor (x_m - x_s) + (y_m - y_s) \rfloor$$
 (16)

Here, $(a, c, d, e, f, \alpha, k) = (40, 5/6, 0.5, 0.65, 20, 20, 300)$. When the error functions are set to $e_x = x_m - x_s$, $e_y = y_m - y_s$, and $e_z = z_m - z_s$, the error dynamics can be written in the form of Eq. (5) with

$$\begin{bmatrix} \dot{e}_{1} \\ \dot{e}_{2} \\ \dot{e}_{3} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} -a & a & 0 & 1 \\ 0 & f & 0 & 1 \\ 0 & 0 & c & 0 \\ -k & -k & 0 & -a \end{bmatrix} \begin{bmatrix} e_{1} \\ e_{2} \\ e_{3} \\ h \end{bmatrix} + \begin{bmatrix} d\left(x_{m}z_{m} - x_{s}z_{s}\right) \\ -x_{m}z_{m} + x_{s}z_{s} \\ x_{m}y_{m} - x_{s}y_{s} - e\left(x_{m}^{2} - x_{s}^{2}\right) \\ 0 \\ 0 \end{bmatrix}.$$
(17)

The characteristic polynomial of matrix \overline{A} in Eq. (17) is given by

$$p(\lambda) = \lambda^{4} + (\alpha + 19.167)\lambda^{3} + (2k + 19.167\alpha - 816.66)\lambda^{2} + (58.334k - 816.66\alpha + 666.4)\lambda + (-49.98k + 666.4\alpha) = 0.$$
⁽¹⁸⁾

According to the Routh–Hurwitz stability criterion, the error dynamics of the systems in Eq. (17) are globally asymptotically stable if the following condition is satisfied:

$$\alpha > 13.9 \land k > 14\alpha - 11.4. \tag{19}$$

Consequently, the proposed advanced ameliorated synchronization scheme successfully synchronized Lü-like chaotic systems. The results are shown in Fig. 1, revealing that the error functions and the controller asymptotically approached zero.

4.2 Circuit simulation

Because the amplitude of the numerical simulation result exceeded the operating voltage of the circuit amplifier, we set the scaling factors of the circuit as $(x_m, y_m, z_m) = (10X_m, 10Y_m, 10Z_m)$, $(x_s, y_s, z_s) = (10X_s, 10Y_s, 10Z_s)$, and h = 10H and set the time scale as $\tau = 10t$ and $(a, c, d, e, f, \alpha, k) = (40, 5/6, 0.5, 0.65, 20, 20, 300)$. Then, the driving system in Eqs. (14)–(16) becomes as:

$$\mathbf{Master}: \begin{cases} \dot{X}_{m} = 400Y_{m} - 400X_{m} + 50X_{m}Z_{m}, \\ \dot{Y}_{m} = 200Y_{m} - 100X_{m}Z_{m}, \\ \dot{Z}_{m} = 8.33Z_{m} + 100X_{m}Y_{m} - 65X_{m}^{2}, \end{cases}$$
(20)

Slave:
$$\begin{cases} X_{s} = 400Y_{s} - 400X_{s} + 50X_{s}Z_{s} - 10H, \\ \dot{Y}_{s} = 200Y_{s} - 100X_{s}Z_{s} - 10H, \\ \dot{Z}_{s} = 8.33Z_{s} + 100X_{s}Y_{s} - 65X_{s}^{2}, \end{cases}$$
(21)



Fig. 1. Time series of (a) $x_m - x_s$, (b) $y_m - y_s$, (c) $z_m - z_s$, and (d) *h* of error dynamics of Lü-like chaotic systems with parameters x_m (0) = y_m (0) = z_m (0) = 0.1, x_s (0) = y_s (0) = z_s (0) = 10, h(0) = 0, α = 20, and coupling strength k = 300 underlying the proposed synchronization scheme based on advanced ameliorated dynamic control.

Dynamic controllers:
$$\dot{H} = -200H - 3000X_m + 3000X_s - 3000Y_m + 3000Y_s$$
. (22)

To transform the above equations into circuits, the systems in Eqs. (20)–(22) were converted to the following systems:

$$\begin{cases} \dot{X}_{m} = \frac{-1}{R_{10}C_{1}} \left(-Y_{m}\right) + \frac{-1}{R_{9}C_{1}} X_{m} + \frac{-1}{R_{11}C_{2}} \frac{X_{m}Z_{m}}{10}, \\ \dot{Y}_{m} = \frac{-1}{R_{12}C_{2}} \left(-Y_{m}\right) + \frac{-1}{R_{13}C_{2}} \frac{X_{m}Z_{m}}{10}, \\ \dot{Z}_{m} = \frac{-1}{R_{14}C_{3}} \left(-Z_{m}\right) + \frac{-1}{R_{15}C_{3}} \frac{-X_{m}Y_{m}}{10} + \frac{-1}{R_{16}C_{3}} \frac{X_{m}X_{m}}{10}, \end{cases}$$
(23)

$$\begin{cases} X_{s} = \frac{-1}{R_{18}C_{4}} \left(-Y_{s}\right) + \frac{-1}{R_{17}C_{4}} X_{s} + \frac{-1}{R_{19}C_{4}} \frac{X_{s}Z_{s}}{10} + \frac{-1}{R_{20}C_{6}} H, \\ \dot{Y}_{s} = \frac{-1}{R_{21}C_{5}} \left(-Y_{s}\right) + \frac{-1}{R_{22}C_{5}} \frac{X_{s}Z_{s}}{10} + \frac{-1}{R_{23}C_{5}} H, \\ \dot{Z}_{s} = \frac{-1}{R_{24}C_{6}} \left(-Z_{s}\right) + \frac{-1}{R_{25}C_{6}} \frac{-X_{s}Y_{s}}{10} + \frac{-1}{R_{26}C_{6}} \frac{X_{s}X_{s}}{10}, \end{cases}$$
(24)

$$\dot{H} = \frac{-1}{R_{27}C_7}H + \frac{-1}{R_{28}C_7}X_m + \frac{-1}{R_{29}C_7}(-X_s) + \frac{-1}{R_{30}C_7}Y_m + \frac{-1}{R_{31}C_7}(-Y_s).$$
(25)

Here, $C_1 = C_1 = \dots C_7 = 10$ nF, $R_9 = R_{10} = R_{17} = R_{18} = 250$ kΩ, $R_{11} = R_{19} = 200$ kΩ, $R_{12} = R_{21} = 10$ kΩ, $R_{13} = R_{15} = R_{22} = R_{25} = 50$ kΩ, $R_{14} = R_{24} = 12000$ kΩ, $R_{16} = R_{26} = 153.8$ kΩ, $R_{20} = R_{23} = 10000$ kΩ, $R_{27} = 500$ kΩ, and $R_{28} = R_{29} = R_{30} = R_{31} = 33$ kΩ. OrCAD software was used to design the above circuit for simulation, and Fig. 2 shows the circuit design of the synchronization scheme of Lü-like chaotic systems based on advanced ameliorated dynamic control, in which the resistors of the inverting amplifiers are set as $R_1 = R_2 = \dots R_8 = 10$ kΩ. Figure 3 shows those of the circuit simulation, which match those of the numerical simulation.

4.3 Circuit implementation

Before implementing the circuit, we designed the PCB using OrCAD Capture and PCB Editor. Figure 4 presents the PCB designs of our synchronization scheme of Lü-like chaotic systems based on advanced ameliorated dynamic control. Then, we operated a circuit engraving machine to print out the circuit board, and we soldered electronic components and connection points on it to complete the real circuits shown in Fig. 5. Signals from the real circuits are displayed on an oscilloscope to visualize the results of synchronization as shown in Fig. 6.

5. Discussion

In 2021, the synchronization scheme based on ameliorated dynamic control was proposed to successfully synchronize Lü-like chaotic systems that could not be synchronized previously. Then, we studied its circuit and successfully simulated it.

Take the following Lü-like chaotic systems as an example.

Master:
$$\begin{cases} \dot{x}_{m} = a(y_{m} - x_{m}) + dx_{m}z_{m} \\ \dot{y}_{m} = fy_{m} - x_{m}z_{m} \\ \dot{z}_{m} = cz_{m} + x_{m}y_{m} - ex_{m}^{2} \end{cases}$$
(26)





Fig. 3. Plots of circuit simulation results. Time series of (a) $x_m - x_s$, (b) $y_m - y_s$, (c) $z_m - z_s$, and (d) h of error dynamics of Lü-like chaotic systems underlying the synchronization scheme based on advanced ameliorated dynamic control.



Fig. 4. (Color online) PCB design diagrams of the synchronization scheme of (a) Lü-like chaotic systems with (b) advanced ameliorated dynamic controller.



Fig. 5. (Color online) Circuit implementation diagrams of the synchronization scheme of (a) Lü-like chaotic systems with (b) advanced ameliorated dynamic controller.



Fig. 6. (Color online) Plots of oscilloscope signals of (a) $x_m - x_s$, (b) $y_m - y_s$, (c) $z_m - z_s$, and (d) *h* of error dynamics of Lü-like chaotic systems from real circuits underlying the synchronization scheme based on advanced ameliorated dynamic control.

Slave:
$$\begin{cases} \dot{x}_{s} = a(y_{s} - x_{s}) + dx_{s}z_{s} \\ \dot{y}_{s} = fy_{s} - x_{s}z_{s} - h_{1} \\ \dot{z}_{s} = cz_{s} + x_{s}y_{s} - ex_{s}^{2} - h_{2} \end{cases}$$
(27)

Dynamic controllers:
$$\begin{cases} \dot{h}_1 = -\alpha h_1 - k \left(y_m - y_s \right) \\ \dot{h}_2 = -\alpha h_2 - k \left(z_m - z_s \right) \end{cases}$$
(28)

When the parameters (a, c, d, e, f, a, k) are (40, 5/6, 0.5, 0.65, 20, 21, 440), Lü-like chaotic systems can achieve chaos synchronization. However, our purpose was to implement the circuit, for which the above scheme was not effective because many electronic components were required. Therefore, we continued to refine the scheme and proposed an advanced ameliorated dynamic controller that combined the two original controllers into one without decreasing the coupling between signals, which was the key to achieving synchronization. Comparing Eqs. (28) and (16), it can be seen that the controller is simplified and the number of electronic components is reduced. The results of the circuit implementation are shown in Fig. 6, which reveals that the error functions are asymptotically stable and the controller asymptotically approaches zero, that is, no additional controller is required when synchronization is achieved.

6. Conclusions

We improved and proposed the synchronization scheme based on advanced ameliorated dynamic control, and we not only performed a circuit simulation but also successfully implemented the circuit. Experimental results verified the feasibility and effectiveness of our proposed synchronization scheme, which could be applied to sensor circuits.

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